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TEMPERATURE SHOCK WAVES IN MAGNETIC HYDRODYNAMICS WITH ACCOUNT FOR THE HYPERBOLIC HEAT TRANSFER (SOLUTION OF THE RUNNING–WAVE TYPE)

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By the method of running waves the solution of magnetic hydrodynamics equations for media with infinite electrical conduction and heat transfer at heat-flow relaxation has been investigated. The characteristics of the considered system of hyperbolic equations have been determined. By means of the theory of generalized solutions of first-order, quasi-linear equations the stability of strong discontinuities of magnetohydrodynamic and thermal values has been proved.

Keywords: *magnetic hydrodynamics with account for the heat transfer, heat-flow relaxation, relations at the strong discontinuity front, stability of discontinuities of magnetohydrodynamic and thermal values.*

Introduction. It is known that the concept that the heat flux is proportional to the temperature gradient (Fourier law) is not always true. For instance, under the action of magnetic pressure on a laser target in theta or zeta pinches and in many other cases the mean free path and time of particles can be determined by the spatial and temporal scales of the change in the temperature of the medium. In these cases, the Fourier law considerably overestimates the values of thermal fluxes, which often leads to incorrect results. The present work deals with the known physically substantiated hyperbolic heat-transfer model taking into account the heat-flow relaxation [1–9]. Solutions of magnetic hydrodynamic equations of the running-wave type are considered. For heat-transfer models based on the Fourier law, it was shown earlier that running-wave-type solutions of gas dynamics equations with account for the heat conductivity of the medium exist only in that region of its parameters where the wave velocity is higher than the velocity of sound [10–13]. Analysis has shown that in the case of heat-flow relaxation its physical properties, as well as the gas-dynamic and thermal parameters can vary over a wide range [8]. Unlike the Fourier model, in the case of hyperbolic heat transfer the functions describing its parameters, including the heat flux and the temperature, can experience a strong discontinuity. In the present work, with the help of the theory of generalized solutions of quasi-linear, first-order equations [14], we prove the stability of the above discontinuities. It has been shown that the character of the running-wave-type solution of magnetic hydrodynamic equations strongly depends on the ratio of the heat-flow relaxation time to the heat-conductivity coefficient of the medium, as well as on the magnetic pressure change before the running wave front.

System of Magnetic Hydrodynamic Equations with Account for the Heat-Flow Relaxation. Consider the magnetic hydrodynamic equations for a medium with planar symmetry and infinite conduction. Let the Lagrange variables be m and t, the gas velocity $v = v(m, t)$, the medium density $\rho = \rho(m, t)$, the material pressure $p = p(m, t)$, the transverse component of the electric field strength $H = H(m, t)$, the temperature of the medium $T = T(m, t)$, the heat-flow density in it $W = W(m, t)$, and the specific internal energy of the medium $\varepsilon = \varepsilon(m, t)$. The equations of continuity, motion, energy, and magnetic field line freezing-in can be written in the form

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=\frac{dv}{dm}\,, \tag{1}$$

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$$\frac{dv}{dt} = -\frac{\partial}{\partial m}\left(p + \frac{H^2}{8\pi}\right), \tag{2}$$

$$\frac{\partial}{\partial t}\left(\varepsilon + \frac{1}{2}v^2 + \frac{H^2}{8\pi\rho}\right) = -\frac{\partial}{\partial m}\left[\left(p + \frac{H^2}{8\pi}\right)v + W\right], \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{H}{\rho}\right) = 0\,. \tag{4}$$

Let us represent the expression of the function W on the right side of Eq. (3) as

$$W = -k\frac{\partial T}{\partial m} - \tau\frac{\partial W}{\partial t}, \tag{5}$$

where $k = k(\rho, T)$ and $\tau = \tau(\rho, t)$. Equations (1)–(5) represent a hyperbolic system. Let us obtain its characteristics by using a number of definitions from the theory of quasi-linear, first-order equations [14] (see also [15]).

Consider the system of equations

$$\frac{\partial \mathbf{u}}{\partial t} + A\frac{\partial \mathbf{u}}{\partial m} = \mathbf{b}\,. \tag{6}$$

Here $\mathbf{u}(m, t) = \{u_1(m, t), ..., u_n(m, t)\}$ is a vector function of the independent variables m and t formed by n sought functions; $A = \{u_i(m, t, \mathbf{u})\}$, $i = 1, 2, ..., n$ is a matrix of order n; $\mathbf{b} = \{b_1(m, t, \mathbf{u}), ..., b_n(m, t, \mathbf{u})\}$ is a vector of the right side of the equation.

Let $\lambda = \lambda(m, t, \mathbf{u})$ be the eigenvalue of the matrix A. From the theory of quasi-linear equations it follows that to find all eigenvalues of λ, it is necessary to solve the equation

$$\det(A - \lambda E) = 0\,. \tag{7}$$

The system of equations (6) is hyperbolic in a certain region of the space m, t, $\mathbf{u}$ if everywhere in this region the eigenvalues $\lambda_1, ..., \lambda_n$ of the matrix A are real and different.

Let us reduce system (1)–(5) at $\tau \neq 0$ to a form analogous to (6). Assuming that the equations of state of the perfect gas

$$p = R\rho T\,, \quad \varepsilon = \frac{R}{\gamma - 1}T\,, \tag{8}$$

where the constant ratio of specific heat capacities $\gamma > 1$, holds, we get

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) - \frac{dv}{dm} = 0\,, \tag{9}$$

$$\frac{\partial v}{\partial t} - R\rho^2 T\frac{\partial}{\partial m}\left(\frac{1}{\rho}\right) + R\rho\frac{\partial T}{\partial m} + \frac{2H}{8\pi}\frac{\partial H}{\partial m} = 0\,, \tag{10}$$

$$\frac{\partial T}{\partial t} + (\gamma - 1)\,\rho T\frac{\partial v}{\partial m} + \frac{\gamma - 1}{R}\frac{\partial W}{\partial m} = 0\,, \tag{11}$$

$$\frac{\partial W}{\partial t} + \frac{k}{\tau}\frac{\partial T}{\partial m} = -\frac{W}{\tau}\,, \tag{12}$$

$$\frac{\partial H}{\partial t} + \rho H \frac{\partial v}{\partial m} = 0\,. \tag{13}$$

In accordance with (6), (9)–(13)

$$\mathbf{u} = \left\{\frac{1}{\rho}, v, T, W, H\right\}, \quad \mathbf{b} = \left\{0, 0, 0, -\frac{W}{\tau}, 0\right\},$$

$$A = \left\{\begin{matrix} 0 & -1 & 0 & 0 & 0 \\ -R\rho^2 T & 0 & R\rho & 0 & 2H/(8\pi) \\ 0 & (\gamma-1)\,\rho T & 0 & (\gamma-1)/R & 0 \\ 0 & 0 & k/\tau & 0 & 0 \\ 0 & \rho H & 0 & 0 & 0 \end{matrix}\right\},$$

and Eq. (7) takes the form

$$\det(A - \lambda E) = \left\{\begin{matrix} -\lambda & -1 & 0 & 0 & 0 \\ -R\rho^2 T & -\lambda & R\rho & 0 & 2H/(8\pi) \\ 0 & (\gamma-1)\,\rho T & -\lambda & (\gamma-1)/R & 0 \\ 0 & 0 & k/\tau & -\lambda & 0 \\ 0 & \rho H & 0 & 0 & -\lambda \end{matrix}\right\} = 0\,,$$

i.e.,

$$\lambda^4 - \lambda^2\left(\rho^2\gamma RT + \rho\frac{H^2}{8\pi} + \frac{(\gamma-1)\,k}{R\tau}\right) + \frac{(\gamma-1)\,k}{R\tau}\left(\rho^2 RT + \rho\frac{H^2}{8\pi}\right) = 0\,. \tag{14}$$

Let us write the mass velocities of sound. Let $C = \rho\sqrt{RT}$, $C_\gamma = \rho\sqrt{\gamma RT} = \sqrt{\gamma}\,C$, $C_h = \rho\frac{H}{\sqrt{4\pi\rho}}$, and $C_T = \sqrt{(\gamma-1)\,k/(R\tau)}$. Let us give Eq. (14) in the biquadratic form

$$\lambda^4 - \lambda^2\left(C_\gamma^2 + C_H^2 + C_T^2\right) + C_T^2\left(C^2 + C_H^2\right) = 0\,. \tag{15}$$

Its solution is

$$\lambda^2 = \frac{1}{2}\left[\left(C_\gamma^2 + C_H^2 + C_T^2\right) \pm \sqrt{\left(C_\gamma^2 + C_H^2 + C_T^2\right)^2 - 4C_T^2\left(C^2 + C_H^2\right)}\,\right]. \tag{16}$$

Note that the radicand in (16) is positive. Therefore, we can write

$$\left(C_\gamma^2 + C_H^2 + C_T^2\right)^2 - 4C_T^2\left(C^2 + C_H^2\right) = \left(C_T^2 - C_\gamma^2 - C_H^2\right)^2 + 4C_T^2\left(C_\gamma^2 - C^2\right) > 0\,.$$

From (16) we obtain the eigenvalues of the matrix A:

$$\lambda = \lambda_1 = \sqrt{\frac{1}{2}\left[C_\gamma^2 + C_H^2 + C_T^2 + \sqrt{\left(C_\gamma^2 + C_H^2 + C_T^2\right)^2 - 4C_T^2\left(C^2 + C_H^2\right)}\,\right]}\,, \quad \lambda = -\lambda_1\,,$$

$$\lambda = \lambda_2 = \sqrt{\frac{1}{2}\left[C_\gamma^2 + C_H^2 + C_T^2 - \sqrt{\left(C_\gamma^2 + C_H^2 + C_T^2\right)^2 - 4C_T^2\left(C^2 + C_H^2\right)}\,\right]}\,, \quad \lambda = -\lambda_2\,. \tag{17}$$

The equalities

$$\frac{dm}{dt} = \lambda_1\,, \quad \frac{dm}{dt} = -\lambda_1\,, \quad \frac{dm}{dt} = \lambda_2\,, \quad \frac{dm}{dt} = -\lambda_2 \tag{18}$$

define four families of characteristics of the system of magnetic hydrodynamic equations (1)–(5) with account for the heat-flow relaxation in the infinite conducting medium.

Relations at the Strong Discontinuity Front of Magnetohydrodynamic and Thermal Quantities. By virtue of the hyperbolicity of the system of equations (1)–(5), the sought functions can experience a strong discontinuity.

Let us introduce an auxiliary function $V = V(m, t)$ satisfying at $\tau \neq 0$ the equation

$$\frac{\partial V}{\partial m} = \frac{k}{\tau} \frac{\partial T}{\partial m} . \tag{19}$$

In view of (19), Eq. (5) at $\tau \neq 0$ takes the form

$$\frac{\partial W}{\partial t} = - \frac{\partial V}{\partial m} - \frac{W}{\tau} . \tag{20}$$

The function $V = V(m, t)$ can be determined from (19) in solving concrete problems.

Let $m = m_{\text{dis}} = m_{\text{dis}}(t)$ be a trace of the discontinuity surface in the plane (m, t) and the mass velocity of discontinuity $D_M = \frac{dm_{\text{dis}}}{dt}$. Let us determine the relations expressing the laws of conservation at the strong discontinuity front, analogously to [16], by means of corresponding integration of each equation of system (1)–(4), (20). Integrating the above equations with respect to the small, contracted to a zero volume, domain of variability of the independent variables m, t, including the discontinuity line, and denoting by "2" the quantities behind the discontinuity front and by "1" those before the front, we get

$$\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right) D_M = v_1 - v_2 , \tag{21}$$

$$(v_2 - v_1)\, D_M = p_2 + \frac{H_2^2}{8\pi} - p_1 - \frac{H_1^2}{8\pi} , \tag{22}$$

$$\left[\frac{R}{\gamma - 1} T_2 + \frac{1}{2} v_2^2 + \frac{H_2^2}{8\pi\rho_2} - \left(\frac{R}{\gamma - 1} T_1 + \frac{1}{2} v_1^2 + \frac{H_1^2}{8\pi\rho_1}\right)\right] D_M$$

$$= \left(p_2 + \frac{H_2^2}{8\pi}\right) v_2 + W_2 - \left(p_1 + \frac{H_1^2}{8\pi}\right) v_1 - W_1 , \tag{23}$$

$$\frac{H_2}{\rho_2} = \frac{H_1}{\rho_1} , \tag{24}$$

$$(W_2 - W_1)\, D_M = V_2 - V_1 . \tag{25}$$

Let us investigate the solution of system (1)–(4), (19), (20) in view of (8) by the running-wave method.

Running-Wave-Type Solutions of Magnetic Hydrodynamic Equations. Formulation of the Problem. Let the velocity of the piston and the temperature of its surface

$$v(0, t) = v_*(t) , \quad T(0, t) = T_*(t) \tag{26}$$

be chosen so that magnetohydrodynamic and thermal perturbations propagate in a medium with parameters

$$v = v_1 = 0\,,\quad T = T_1 \geq 0\,,\quad \rho = \rho_1 = \rho_0\,,\quad W = W_1 = 0\,,\quad H = H_1 > 0\,,\quad V = V_1 \tag{27}$$

in the form of a running wave. This means that each function $F = F(m, t)$ satisfying system (1)–(4), (19), (20) is representable in the form $F(m, t) = F(Dt - m)$, $D = \text{const}$.

Let

$$k = k_0 T^{a_0} \rho^{b_0}\,,\quad \tau = \tau_0 T^{a_1} \rho^{b_1}\,,\quad a_0 \geq a_1 > 0\,. \tag{28}$$

Let us represent the independent variables and sought functions in the form

$$x = \frac{Dt - m}{M_0}\,,\quad \eta = \eta\,(x) = \frac{\rho_0}{\rho\,(m, t)}\,,\quad \alpha = \alpha\,(x) = \frac{v\,(m, t)}{D\rho_0^{-1}}\,,$$

$$f = f\,(x) = \frac{RT\,(m, t)}{D^2 \rho_0^{-2}}\,,\quad \beta = \beta\,(x) = \frac{p\,(m, t)}{D^2 \rho_0^{-1}}\,,\quad h = h\,(x) = \frac{H\,(m, t)}{D\rho_0^{-0.5}}\,, \tag{29}$$

$$\omega = \omega\,(x) = \frac{W\,(m, t)}{D^3 \rho_0^{-2}}\,,\quad \tilde{V} = \tilde{V}\,(x) = \frac{V\,(m, t)}{D^4 \rho_0^{-2}}\,.$$

where $M_0 = \{k_0 D^{2a_0 - 1} R^{-(a_0+1)} \rho_0^{-2a_0 + b_0}$ is a constant.

Let us reduce system (1)–(4), (19), (20) with the aid of (29) in view of (8) and (28) to the following ordinary differential equations for x:

$$\frac{d\eta}{dx} = -\frac{d\alpha}{dx}\,,\quad \frac{d\alpha}{dx} = \frac{d}{dx}\left(\beta + \frac{h^2}{8\pi}\right),\quad \frac{d}{dx}\,(h\eta) = 0\,,$$

$$\frac{d}{dx}\left(\frac{1}{\gamma - 1} f + \frac{1}{2}\alpha^2 + \frac{h^2}{8\pi}\eta\right) = \frac{d}{dx}\left[\left(\beta + \frac{h^2}{8\pi}\right)\alpha + \omega\right], \tag{30}$$

$$\frac{d\tilde{V}}{dx} = \frac{1}{\varphi_0} f^{a_0 - a_1} \eta^{-b_0 + b_1} \frac{df}{dx}\,, \tag{31}$$

$$\frac{d\omega}{dx} = \frac{d\tilde{V}}{dx} - \frac{1}{\varphi_0}\,\omega f^{-a_1} \eta^{b_1}\,. \tag{32}$$

Here $\beta = f/\eta$ and $\varphi_0 = \dfrac{\tau_0 R^{a_0 - a_1 + 1}}{k_0 D^{2(a_0 - a_1 - 1)} \rho_0^{-2(a_0 - a_1) + (b_0 - b_1)}}$ is a dimensionless constant. Integrating Eqs. (30), we get

$$\eta = C_0 - \alpha\,,\quad \alpha = C_1 + \beta + \frac{h^2}{8\pi}\,,\quad h\eta = C_2\,,\quad \frac{1}{\gamma - 1} f + \frac{1}{2}\alpha^2 + \frac{h^2}{8\pi}\eta - \left(\beta + \frac{h^2}{8\pi}\right)\alpha - \omega = C_3\,, \tag{33}$$

where C_0, C_1, C_2, and C_3 are constants.

Define the magnetic pressure by the formula $p_H = H^2/8\pi$.

Assume that $D_M = D$. In the variables of (29), relations (21)–(25), (27) will take the form

$$\alpha = \alpha_1 = 0\,,\quad f = f_1 \geq 0\,,\quad \eta = \eta_1 = 1\,,\quad \omega = \omega_1 = 0\,,\quad h = h_1 > 0\,,\quad \tilde{V} = \tilde{V}_1\,; \tag{34}$$

$$\eta_2 - 1 = -\alpha_2\,,\quad \alpha_2 = \beta_2 + \beta_{H_2} - f_1 - \beta_{H_1}\,,\quad h_2\eta_2 = h_1\,,\quad \omega_2 = \tilde{V}_2 - \tilde{V}_1\,,$$

$$\frac{1}{\gamma-1} f_2 + \frac{1}{2}\alpha_2^2 + \beta_{H_2}\eta_2 - \frac{1}{\gamma-1} f_1 - \beta_{H_1} = (\beta_2 + \beta_{H_2})\,\alpha_2 + \omega_2\,, \tag{35}$$

where $\beta_H = h^2/8\pi$; $\beta_1 = f_1$.

At the running wave front, let a strong discontinuity of the sought functions take place. In this case, the constants entering into (33) will have the form

$$C_0 = 1\,,\quad C_1 = -f_1 - \beta_{H_1}\,,\quad C_2 = h_1\,,\quad C_3 = \frac{1}{\gamma-1} f_1 + \beta_{H_1}\,. \tag{36}$$

In view of (36) and (33), by manipulations we get

$$\alpha = 1 - \eta\,,\quad \beta = f_1 + 1 - \eta - \beta_{H_1}\eta^{-2}(1-\eta^2)\,,\quad f = \eta\,(f_1 + 1 - \eta) - \beta_{H_1}\eta^{-1}(1-\eta^2)\,,$$

$$h = h_1\eta^{-1}\,,\quad \omega = \frac{1}{2}\frac{\gamma+1}{\gamma-1}(1-\eta)\left[\eta - \frac{\gamma-1}{\gamma+1} - \frac{2\gamma}{\gamma+1} f_1 - \frac{2}{\gamma+1}\beta_{H_1}((2-\gamma)\,\eta^{-1} + \gamma)\right]. \tag{37}$$

Let us represent formulas (35) as

$$\alpha_2 = 1 - \eta_2\,,\quad \beta_2 = f_1 + 1 - \eta_2 - \beta_{H_1}\eta_2^{-2}(1-\eta_2^2)\,,\quad f_2 = \eta_2\,(f_1 + 1 - \eta_2) - \beta_{H_1}\eta_2^{-1}(1-\eta_2^2)\,,$$

$$h_2 = h_1\eta_2^{-1}\,,\quad \omega_2 = \frac{1}{2}\frac{\gamma+1}{\gamma-1}(1-\eta_2)\left[\eta_2 - \frac{\gamma-1}{\gamma+1} - \frac{2\gamma}{\gamma+1} f_1 - \frac{2}{\gamma+1}((2-\gamma)\,\eta_2^{-1} + \gamma)\,\beta_{H_1}\right], \tag{38}$$

$$\tilde{V}_2 = \tilde{V}_1 + \omega_2\,.$$

Note that expressions (37) at the running wave front satisfy both relations (38) (a strong discontinuity takes place) and directly conditions (34) (in the vicinity of the front the functions are continuous).

Let us obtain the equation defining the dimensionless function of the specific volume $\eta = \eta(x)$. Using formulas (31), (32), and (37), we can write:

$$\frac{d\eta}{dx} = \frac{1}{2}\frac{\gamma+1}{\gamma-1}(1-\eta)\,\eta^{a_0+b_0+1}\left[\eta^2 - \frac{1}{\gamma+1}(\gamma - 1 + 2\gamma\,(f_1 + \beta_{H_1}))\,\eta - \frac{2}{\gamma+1}\beta_{H_1}(2-\gamma)\right]\frac{1}{\Delta}\,, \tag{39}$$

where

$$\Delta = \left[\eta^2 (f_1 + 1 - \eta) - \beta_{H_1}(1-\eta^2)\right]^{a_0}\left[(f_1 + 1 - 2\eta)\,\eta^2 + \beta_{H_1}(1+\eta^2)\right]$$

$$+ \varphi_0\frac{\gamma+1}{\gamma-1}\eta^{a_0 - a_1 + b_0 - b_1}\left[\eta^2\left(\eta - \frac{\gamma}{\gamma+1}(f_1+1)\right) - \frac{1}{\gamma+1}\beta_{H_1}(2 - \gamma + \gamma\eta^2)\right]$$

$$\times\left[\eta^2(f_1 + 1 - \eta) - \beta_{H_1}(1-\eta^2)\right]^{a_1}. \tag{40}$$

The function $\tilde{V} = \tilde{V}(x)$ satisfies the equation

$$\frac{d\tilde{V}}{d\eta} = \frac{1}{\varphi_0}\eta^{a_0 - a_1 - b_0 + b_1}\left[f_1 + 1 - \eta - \beta_{H_1}(\eta^{-2} - 1)\right]^{a_0 - a_1}\left[f_1 + 1 - 2\eta + \beta_{H_1}(\eta^{-2} + 1)\right]. \tag{41}$$

To the running wave front, let there correspond the relation $m = Dt$, which in the variables of (29) is adequate to the value of $x = 0$. The perturbed medium is in the domain $m \le Dt$, i.e., at $x \ge 0$. Note the following fact. From the gas dynamics it is known at $\varphi_0 = 0$ (heat transfer follows the Fourier law $W = -k\frac{\partial T}{\partial m}$) the solution of Eq. (39) is continuous. In so doing, the boundary condition (34) at the wave front is fulfilled at a finite value of x if $f_1 = 0$. Since in (39) the shear transformation $x = x' + \text{const}$ is admissible, the position of the running wave front can be fixed by the coordinate $x = 0$. In the case where $f_1 \neq 0$, the sought functions safisty condition (34) at $x = -\infty$ [10–12]. However, if at the wave front a strong discontinuity takes place, then at any $f_1 \ge 0$ the boundary conditions (34), (38) can be formulated at a finite value of x and, in particular, at $x = 0$.

To the gas-piston boundary in the initial Lagrange variables m, t there corresponds the value of $m = 0$. Then from (29) we obtain $x = x_*$, where $x_* = \frac{D}{m_0}t$. The dimensionless function $\eta = \eta_* = \eta(x_*)$ corresponds to the specific volume in the plane of $m = 0$, and $\alpha(x_*)$ and $f(x_*)$ correspond to the velocity and the temperature. This means that the solution of Eq. (39) determines both the profiles of the sought quantities describing the propagation of the running wave in the region of $x \ge 0$ and the distributions of functions (26) creating this wave at $t \ge 0$. Using (29), (37), we obtain

$$v_*(t) = \alpha(x_*)\, D\rho_0^{-1} = (1 - \eta(x_*))\, D\rho_0^{-1},$$

$$T_*(t) = f(x_*)\, R^{-1} D^2 \rho_0^{-2} = (1 - \eta(x_*))\,(\eta(x_*) - \beta_{H_1}\eta(x_*)^{-1}(1 - \eta(x_*)^2))\, R^{-1} D^2 \rho_0^2 .$$

Characteristic Properties of the Solution. Let at some value of $\eta = \eta_0$, $0 < \eta_0 \le 1$ the relation

$$\Delta(\eta_0) = 0 . \tag{42}$$

hold. The derivative $\left.\frac{d\eta}{dx}\right|_{\eta=\eta_0}$ is infinite. Let us then have at $\eta = \eta_{\gamma_0}$

$$\eta_{\gamma_0}^2 - \frac{1}{\gamma+1}\left[\gamma - 1 + 2\gamma(\beta_{H_1} + f_1)\right]\eta_{\gamma_0} - \frac{2}{\gamma+1}\beta_{H_1}(2-\gamma) = 0 . \tag{43}$$

The derivative $\left.\frac{d\eta}{dx}\right|_{\eta=\eta_{\gamma_0}}$ is equal to zero.

In the gas dynamics ($h \equiv 0$, $\beta_{H_1} = 0$) in the case where $\varphi_0 = 0$, the function $\eta = \eta(x)$ satisfies the boundary condition $\eta(0) = 1$. At $f_1 = 0$, from (42), (43) we get $\eta_0 = 0.5$, $\eta_{\gamma_0} = \frac{\gamma-1}{\gamma+1}$. Analysis shows [10–12] that if $1 < \gamma < 3$, then $0 < \eta_{\gamma_0} < \eta_0 < 1$. The value of $\eta = \eta_0$ is attained at some $x = x_0 > 0$. The coordinates of $x = x_0$, $\eta = \eta_0$ in the plane (x, η) characterize the turning point of the integral curves of Eq. (39). At $\eta \to \eta_{\gamma_0}$ we obtain $x \to -\infty$. The solution has a physical meaning only in the region of $\eta_0 \le \eta \le 1$, for x varying over the range $0 \le x \le x_0$, and, consequently, in the finite time interval $0 \le t \le t(x_0)$. If $\varphi_0 > 0$, then, depending on the change in the parameter φ_0 both at $\eta(0) = 1$ and in the presence of a strong discontinuity at the running wave front $\eta(0) = \eta_2 < 1$, a solution of the problem can exist at $0 \le x \le \infty$ (see [8]).

Consider now the possible character of the solution under consideration for the case where $h \neq 0$, $\beta_{H_1} > 0$.

The positive root of Eq. (43) is defined by the formula

$$\eta = \eta_{\gamma_0} = \frac{1}{\gamma+1}\left\{0.5\,(\gamma-1) + \gamma\,(f_1 + \beta_{H_1}) + \sqrt{[0.5\,(\gamma-1) + \gamma\,(f_1 + \beta_{H_1})]^2 + 2\,(2-\gamma)\,(\gamma+1)\,\beta_{H_1}}\right\}. \tag{44}$$

Consider the solution of the problem on the assumption that the value of the parameter η_{γ_0} is in the range of $0 < \eta_{\gamma_0} < \eta(0) \le 1$. The condition $\eta_{\gamma_0} \le 1$ leads to the inequality

$$1 - \gamma f_1 - 2\beta_{H_1} \ge 0\,.$$

Consider the solution of Eqs. (39), (40) in the vicinity of $\eta(0) = 1$. Assume that $\eta = 1 - \tilde{\eta}$, where $\tilde{\eta} > 0$ is a small quantity. Let $f_1 \ge 0$. Preserving the principal terms, we can give Eq. (39) in the form

$$\frac{d\tilde{\eta}}{dx} = \frac{(1 - 2\beta_{H_1} - \gamma f_1)\,\tilde{\eta}}{(\gamma-1)\,(1 - 2\beta_{H_1} - \gamma f_1)\,[f_1 + (1 - 2\beta_{H_1})\,\tilde{\eta}]^{a_0} - \varphi_0\,(1 - 2\beta_{H_1} - \gamma f_1)\,[f_1 + (1 - 2\beta_{H_1})\,\tilde{\eta}]^{a_1}}\,. \tag{45}$$

If $f_1 > 0$, then the solution of Eq. (45) in the vicinity of $\eta = 1$ is defined by the function $\tilde{\eta} = \text{const}\,\exp\,(A_1 x)$, where the constant A_1 is expressed in terms of the parameters γ, φ_0, f_1, and β_{H_1}. The value of $\eta = 1$ is attained at $x = \pm\infty$.

Consequently, at $f_1 > 0$ the wave front can be characterized by the finite value of $x = 0$ if we assume the presence at the front of a strong discontinuity of the sought quantities. In so doing, we should have $\eta(0) = \eta_2 < 1$.

At $f_1 = 0$, Eq. (45) takes the form

$$\left[(\gamma-1)\,\varphi_1^{a_0}\tilde{\eta}^{\,a_0-1} - \varphi_0\varphi_1^{a_1}\tilde{\eta}^{\,a_1-1}\right]\frac{d\tilde{\eta}}{dx} = 1\,, \tag{46}$$

where $\varphi_1 = 1 - 2\beta_{H_1} > 0$. If $a_0 > a_1 > 0$, then the principal term on the left side of Eq. (46) is the second term. The approximate solution under the condition $\tilde{\eta} = 0$ has the form

$$-\varphi_0\varphi_1^{a_1}\tilde{\eta}^{\,a_1} = a_1 x\,. \tag{47}$$

However, relation (47) has no physical meaning, since the left and the right sides contain different signs. Consequently, in the case of $f_1 = 0$, $a_0 > a_1 > 0$, as at $f_1 > 0$ and any positive values of the parameters a_0, a_1, and φ_0, the solution should have at the running wave front a strong discontinuity of the sought quantities.

Let $a_0 = a_1 > 0$, $\varphi_0 \ne \gamma - 1$. Integrating (46) in the vicinity of $x = 0$, $\tilde{\eta} = 0$, we obtain

$$\varphi_1^{a_0}\tilde{\eta}^{\,a_0} = \frac{a_0}{\gamma - 1 - \varphi_0}\,x\,. \tag{48}$$

Relation (48) holds at $0 \le \varphi_0 < \gamma - 1$. The sought functions in the vicinity of $x = 0$, $\eta = 1$ are continuous. From (48) we get

$$\eta = 1 - \left[\frac{a_0}{(1 - 2\beta_{H_1})\,(\gamma - 1 - \varphi_0)}\right]^{\frac{1}{a_0}} x^{\frac{1}{a_0}}\,.$$

Analysis shows that the solution is also continuous at $\varphi_0 = \gamma - 1$. In this case, $\tilde{\eta} = x^{1/(a_0+1)}$. In the case where $\varphi_0 > \gamma - 1$, at the running wave front a strong discontinuity takes place.

Consider the asymptotic solution of Eq. (39) in the vicinity of $\eta = \eta_{\gamma_0}$, assuming $0 < \eta_{\gamma_0} < 1$. Assume $\eta = \eta_{\gamma_0} + z$, where $z > 0$ is a small quantity. Upon transformations to an accuracy of the principal terms we can write (39) in the form

$$\frac{dz}{dx} = \frac{(1-\eta_{\gamma_0})\,\eta_{\gamma_0}^{a_0+b_0+1}}{(\gamma-1)\,\Delta_{\gamma_0}} \sqrt{[0.5\,(\gamma-1)+\gamma\,(f_1+\beta_{H_1})]^2 + 2\,(2-\gamma)\,(\gamma+1)\,\beta_{H_1}\,z}\,. \tag{49}$$

Let us introduce the notation

$$L_0 = \eta_{\gamma_0}^2\,(f_1 + 1 - \eta_{\gamma_0}) - \beta_{H_1}\,(1-\eta_{\gamma_0}^2)\,, \quad L_1 = (f_1 + 1 - 2\eta_{\gamma_0})\,\eta_{\gamma_0}^2 + \beta_{H_1}\,(1+\eta_{\gamma_0}^2)\,,$$

$$L_2 = \frac{\gamma+1}{\gamma-1}\,\eta_{\gamma_0}^{a_0-a_1+b_0-b_1}\left[\frac{1}{\gamma+1}\,\beta_{H_1}\,(2-\gamma+\gamma\eta_{\gamma_0}^2) - \eta_{\gamma_0}^2\left(\eta_{\gamma_0} - \frac{\gamma}{\gamma+1}\,(f_1+1)\right)\right],$$

where $L_0 > 0$ and L_1, and L_2 are other than zero. The relation $\Delta_{\gamma_0} = \Delta(\eta_{\gamma_0})$ can be written in the form $\Delta_{\gamma_0} = L_0^{a_0} L_1 - \varphi_0 L_2 L_0^{a_1}$. Let us consider $\Delta_{\gamma_0} \neq 0$. Assume

$$\Phi_0 = \frac{\Delta_{\gamma_0}\,(\gamma-1)}{(1-\eta_{\gamma_0})\,\eta_{\gamma_0}^{a_0+b_0+1}\sqrt{[0.5\,(\gamma-1)+\gamma\,(f_1+\beta_{H_1})]^2 + 2\,(2-\gamma)\,(\gamma+1)\,\beta_{H_1}}}\,.$$

Let us represent Eq. (49) as

$$\frac{dz}{dx} = \frac{1}{\Phi_0}\,z\,. \tag{50}$$

The solution of (50) has the form

$$x = \Phi_0 \ln Cz\,, \tag{51}$$

where C is a positive constant. From (51) it follows that at $\Phi_0 > 0$ ($L_2\Phi_0 < L_0^{a_0-a_1} L_1$) we have $\lim_{z\to 0} x = -\infty$. This means that with a change in the region of $\eta_{\gamma_0} \le \eta \le 1$, $x \ge 0$ the function $\eta = \eta(x)$ distribution changes direction. Analogously to the case of $h \equiv 0$, a solution of the running-wave type exists only on the finite interval $1 \ge \eta \ge \eta_1$, $0 \le x \le x_1$, where $\eta_{\gamma_0} < \eta_1 < 1$, and the coordinates $x = x_1$, $\eta = \eta_1$ characterize the turning point of the integral curve $\eta = \eta(x)$. The derivative $\frac{d\eta}{dx}$ at $x = x_1$, $\eta = \eta_1$ is infinite.

In the case of $\Phi_0 < 0$ ($L_2\Phi_0 > L_0^{a_0-a_1} L_1$), we get $\lim_{z\to 0} x = +\infty$. A running-wave-type solution exists throughout the range of variability of $x \ge 0$ and, consequently, of the independent variables m and t. In particular, at $L_1 > 0$, $L_2 > 0$ (e.g., $\eta_{\gamma_0} < \frac{1}{2}(f_1 + 1)$) the character of the distribution of the function $\eta = \eta(x)$ and of the other unknown quantities is defined by the inequality $\Phi_0 > L_0^{a_0-a_1} L_1 / L_2$. Thus, in the case where hyperbolic heat transfer takes place, the solution can exist at $0 \le x \le \infty$, $\eta_2 \ge \eta \ge \eta_{\gamma_0}$, where $0 < \eta_1 < 1$. An important role is played also by the change in the magnetic pressure $\beta_{H_1} = h_1^2/(8\pi)$ specified at the initial background of (34) ahead of the running wave front.

Stability of Discontinuous Solutions. The stability of strong discontinuities of the temperature and heat-flow functions at $\tau \neq 0$, $H \equiv 0$ in the case of a stagnant medium was considered in [3, 17]. In these works, the stability was analyzed from the point of view of the theory of generalized solutions of the system of quasi-linear equations of [14]. A similar method is also applicable for investigating the stability of discontinuous solutions of the system of magnetic hydrodynamic equations (1)–(5) at $\tau \neq 0$, $H \neq 0$, $\varphi > 0$ with the example of running waves.

Let a strong discontinuity take place at the running wave front. In the variables of (29), the sought functions satisfy the boundary conditions (34), (38). The squared mass velocities of sound can be given as

$$C^2 = D^2\eta^{-2}f, \quad C_\gamma^2 = \gamma C^2 = \gamma D^2\eta^{-2}f, \quad C_T^2 = (\gamma-1)\frac{k}{R\tau} = \frac{\gamma-1}{\varphi_0}D^2 f^{a_0-a_1}\eta^{-b_0+b_1},$$
$$C_H^2 = \rho^2\frac{H^2}{4\pi\rho} = \frac{D^2}{4\pi}\eta^{-1}h^2. \tag{52}$$

Assume

$$a_0 - a_1 = 1, \quad b_0 = 1, \quad b_1 = -1. \tag{53}$$

In particular, conditions (53) are satisfied by the coefficients k and τ written for completely ionized plasma [18]: $k = k_0 T^{5/2}\rho$; $\tau = \tau_0 T^{3/2}\rho^{-1}$. The squared velocity of sound C_T in the case of (53) has the form $C_T^2 = \frac{\gamma-1}{\varphi_0}D^2 f\eta^{-2}$. In view of (52), (53), we obtain the following expressions of eigenvalues (17):

$$\lambda = \lambda_1 = D\eta^{-1}\sqrt{\frac{1}{2}\left[\left(\gamma+\frac{\gamma-1}{\varphi_0}\right)f + \frac{h^2}{4\pi}\eta + \sqrt{\left[\left(\gamma+\frac{\gamma-1}{\varphi_0}\right)f+\frac{h^2}{4\pi}\eta\right]^2 - 4\frac{\gamma-1}{\varphi_0}f\left(f+\frac{h^2}{4\pi}\eta\right)}\right]},$$
$$\lambda = \lambda_2 = D\eta^{-1}\sqrt{\frac{1}{2}\left[\left(\gamma+\frac{\gamma-1}{\varphi_0}\right)f + \frac{h^2}{4\pi}\eta - \sqrt{\left[\left(\gamma+\frac{\gamma-1}{\varphi_0}\right)f+\frac{h^2}{4\pi}\eta\right]^2 - 4\frac{\gamma-1}{\varphi_0}f\left(f+\frac{h^2}{4\pi}\eta\right)}\right]}, \tag{54}$$
$$\lambda = -\lambda_1, \quad \lambda = -\lambda_2.$$

In the theory of solving systems of n-quasi-linear, first-order equations, the stability conditions of a strong discontinuity are formulated as follows [14] (see also [3, 17]): $n+1$ incoming characteristics should arrive at each point of the discontinuity trajectory $m_{dis} = m_{dis}(t)$ and, accordingly, $n-1$ characteristics should leave it. Let N be a point on the discontinuity line. The law of motion of the discontinuity front is defined by the formula

$$\frac{dm_{dis}}{dt} = D. \tag{55}$$

Consider the characteristics corresponding to the values of the sought quantities behind the front and before the discontinuity front. Assume

$$A_2 = \left(\gamma + \frac{\gamma-1}{\varphi_0}\right)f_2 + 2\beta_{H_1}\eta_2^{-1}, \quad B_2 = \sqrt{A_2^2 - 4\frac{\gamma-1}{\varphi_0}f_2(f_2 + 2\beta_{H_1}\eta_2^{-1})} \tag{56}$$

and, accordingly,

$$A_1 = \left(\gamma + \frac{\gamma-1}{\varphi_0}\right)f_1 + 2\beta_{H_1}, \quad B_1 = \sqrt{A_1^2 - 4\frac{\gamma-1}{\varphi_0}f_1(f_1 + 2\beta_{H_1})}. \tag{57}$$

Using (56) and (57), give the eigenvalues of (56) in the form

$$\lambda = \lambda_{1,2} = D\eta_2^{-1}\sqrt{\frac{1}{2}(A_2+B_2)}, \quad \lambda = \lambda_{2,2} = D\eta_2^{-1}\sqrt{\frac{1}{2}(A_2-B_2)}, \quad \lambda = -\lambda_{1,2}, \quad \lambda = -\lambda_{2,2};$$
$$\lambda = \lambda_{1,1} = D\sqrt{\frac{1}{2}(A_1+B_1)}, \quad \lambda = \lambda_{2,1} = D\sqrt{\frac{1}{2}(A_1-B_1)}, \quad \lambda = -\lambda_{1,1}, \quad \lambda = -\lambda_{2,1}. \tag{58}$$

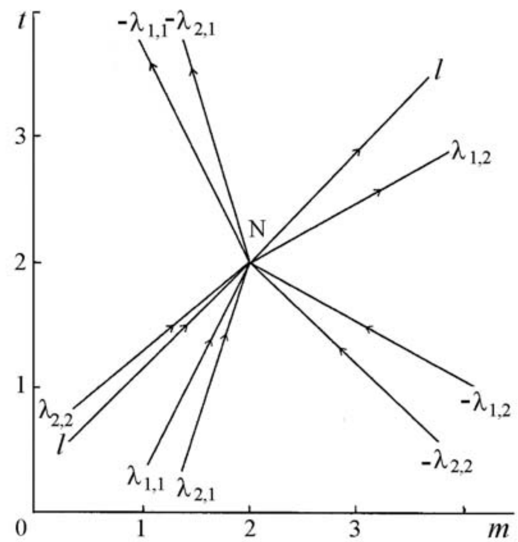

Fig. 1. Characteristics of (59) intersecting the discontinuity line $m_{\text{dis}} = m_{\text{dis}}(t)$ at point N.

In the formulas of (58), in the expressions $\lambda = \pm\lambda_{j,k}$ the index j corresponds to the eigenvalue of $\pm\lambda j$ (j = 1, 2), and the index k — to the values of the sought functions ahead of the discontinuity front (k = 1) and behind the discontinuity front (k = 2).

Using (18), (58), we obtain equalities defining the characteristics in the vicinity of the point N on the discontinuity line $m = m_{\text{dis}}$:

$$\frac{dm}{dt} = \lambda_{1,1}\,,\quad \frac{dm}{dt} = -\,\lambda_{1,1}\,,\quad \frac{dm}{dt} = \lambda_{2,1}\,,\quad \frac{dm}{dt} = -\,\lambda_{2,1}\,,$$
$$\frac{dm}{dt} = \lambda_{1,2}\,,\quad \frac{dm}{dt} = -\,\lambda_{1,2}\,,\quad \frac{dm}{dt} = \lambda_{2,2}\,,\quad \frac{dm}{dt} = -\,\lambda_{2,2}\,. \tag{59}$$

The values of the dimensionless functions of the temperature $f = f_2$ and specific volume $\eta = \eta_2$ behind the discontinuity front and of the quantities $f = f_1$, $\eta = 1$, and $\beta_H = \beta_{H_1}$ ahead of the front satisfy relations (34), (38). We obtained the expression for the function $\tilde{V} = \tilde{V}(x)$ by solving Eq. (41). At $a_0 - a_1 = 1$, $b_1 = -1$ this equation has the form

$$\frac{d\tilde{V}}{d\eta} = \frac{1}{\varphi_0}\,\eta^{-1}\left[f_1 + 1 - \eta - \beta_{H_1}(\eta^{-2} - 1)\right]\left[f_1 + 1 - 2\eta + \beta_{H_1}(\eta^{-2} + 1)\right],\quad \varphi_0 = R^2\tau_0/k_0\,. \tag{60}$$

Integrating (60) at the boundary conditions $\eta = 1$, $\tilde{V} = \tilde{V}_1$, we obtain

$$\tilde{V} = \tilde{V}_1 + \frac{1}{\varphi_0}\left\{(1-\eta)\left[3f_1 + 2 + 3\beta_{H_1} - \eta - \beta_{H_1}\eta^{-1} + \frac{1}{4}\beta_{H_1}^2\eta^{-4}(1+\eta)(1+\eta^2)\right]\right.$$
$$\left. + (f_1 + 1 + \beta_{H_1})^2 \ln\eta\right\}. \tag{61}$$

Using (38) and (61) at specified values of the quantities γ, φ_0, f_1, and β_{H_1}, we determine the parameter $\eta = \eta_2$, $0 < \eta_2 < 1$, from the transcendent equation

$$\tilde{V}_2 - \tilde{V}_1 = \omega_2 = \frac{1}{2}\frac{\gamma+1}{\gamma-1}(1-\eta_2)\left[\eta_2 - \frac{\gamma-1}{\gamma+1} - \frac{2\gamma}{\gamma+1}f_1 - \frac{2}{\gamma+1}(2-\gamma+\gamma\eta_2)\,\beta_{H_1}\eta_2^{-1}\right].$$

The characteristics of (59) were calculated for the values of $\gamma = \frac{5}{3}$, $a_0 - a_1 = 1$, $b_0 = 1$, $b_1 = -1$, $D = 1$, and $\eta_2 = 0.5$ and, accordingly, for $f_1 = \beta_1 = 0.05$, $\varphi_0 = \frac{\tau_0}{k_0}R^2 = 0.2067$, and $\beta_{H_1} = 0.05$. Behind the discontinuity front, we get $f_2 = 0.2$, $\beta_2 = 0.4$, and $\beta_{H_2} = 0.2$.

Analysis shows that in the considered case where $n = 4$, $n+1 = 5$ characteristics arrive at the point N and $n-1 = 3$ characteristics leave it (see Fig. 1). This means that the strong discontinuities of the sought functions at the running wave front $x = 0$ are stable.

Computing Experiments. Equation (39) is solved numerically in the region of $x \ge 0$, $0 < \eta \le \eta(0) \le 1$.

Assume that

$$f_1 = 0\,,\quad a_0 = a_1 = 1\,,\quad b_0 = b_1 = 1\,. \tag{62}$$

Equation (39) therewith will take the form

$$\frac{d\eta}{dx} = \frac{1}{2}\eta^3\left[(\gamma+1)\,\eta^2 - (\gamma - 1 + 2\gamma\beta_{H_1})\,\eta - 2\beta_{H_1}(2-\gamma)\right]\left\{[\eta^2 - \beta_{H_1}(\eta+1)]\right.$$
$$\left.\times\left\{(\gamma-1)\,[(1-2\eta)\,\eta^2 + \beta_{H_1}(1+\eta^2)] + \varphi_0\,[\eta^2\,((\gamma+1)\,\eta - \gamma) - \beta_{H_1}(2-\gamma+\gamma\eta^2)]\right\}\right\}^{-1}. \tag{63}$$

It will be recalled that the constant φ_0 in (41), (63) is defined by the formula

$$\varphi_0 = \tau_0 R^{a_0-a_1+1}\Big/\left[k_0 D^{2(a_0-a_1-1)}\,\rho_0^{-2(a_0-a_1)+b_0-b_1}\right].$$

It has been shown in the foregoing that in the case of $f_1 = 0$, $a_0 = a_1$, at $\varphi_0 < \gamma - 1$ the function $\eta = \eta(x)$ at the running wave front, where $x = 0$, satisfies directly the condition $\eta(0) = 1$. The solution of the problem is continuous.

If $\varphi_0 > \gamma - 1$, then at the front x = 0 a strong discontinuity of the sought quantities takes place. The solution of Eq. (41) for the case (62) $\left(\frac{d\tilde{V}}{d\eta} = \frac{1}{\varphi_0}(1 - 2\eta + \beta_{H_1}(\eta^{-2}+1))\right)$ at the boundary condition $\eta(0) = 1$, $\tilde{V}(0) = \tilde{V}_1$ has the form

$$\tilde{V} = \tilde{V}_1 + \frac{1}{\varphi_0}(1-\eta)\,(\eta - \beta_{H_1}\eta^{-1}(1+\eta))\,. \tag{64}$$

Let us reduce, with the aid of (38) and (64), the relations at the discontinuity front $\tilde{V}_2 - \tilde{V}_1 = \omega_2$ to the quadratic equation

$$\left(\frac{\gamma+1}{\gamma-1}\varphi_0 - 2\right)\eta_2^2 - \left[\varphi_0\left(1 + \frac{2\gamma}{\gamma-1}\beta_{H_1}\right) - 2\beta_{H_1}\right]\eta_2 - \left(\frac{2\,(2-\gamma)}{\gamma-1}\varphi_0 - 2\right)\beta_{H_1} = 0\,. \tag{65}$$

From (65) we determine $\eta(0) = \eta_2$, $0 < \eta_2 < 1$, at specified values of γ, φ_0, and β_{H_1}.

Figure 2 shows the distribution of the function $\eta = \eta(x)$, $x \ge 0$, $0 < h \le 1$ at $\gamma = 5/3$ and various values of the magnetic pressure β_{H_1} before the running wave front and of the dimensionless constant φ_0, which in the case (62) is defined by the formula $\varphi = \frac{\tau_0}{k_0}RD^2$.

Calculations show that depending on the change in the parameters φ_0 and β_{H_1} a solution of Eq. (63) can exist both on the finite interval $0 \le x \le x_1$, where $0 \le m \le m_1(x_1)$, $0 \le t \le t_1(x_1)$, and at $0 \le x \le \infty$, i.e., at $m \ge 0$, $t \ge 0$.

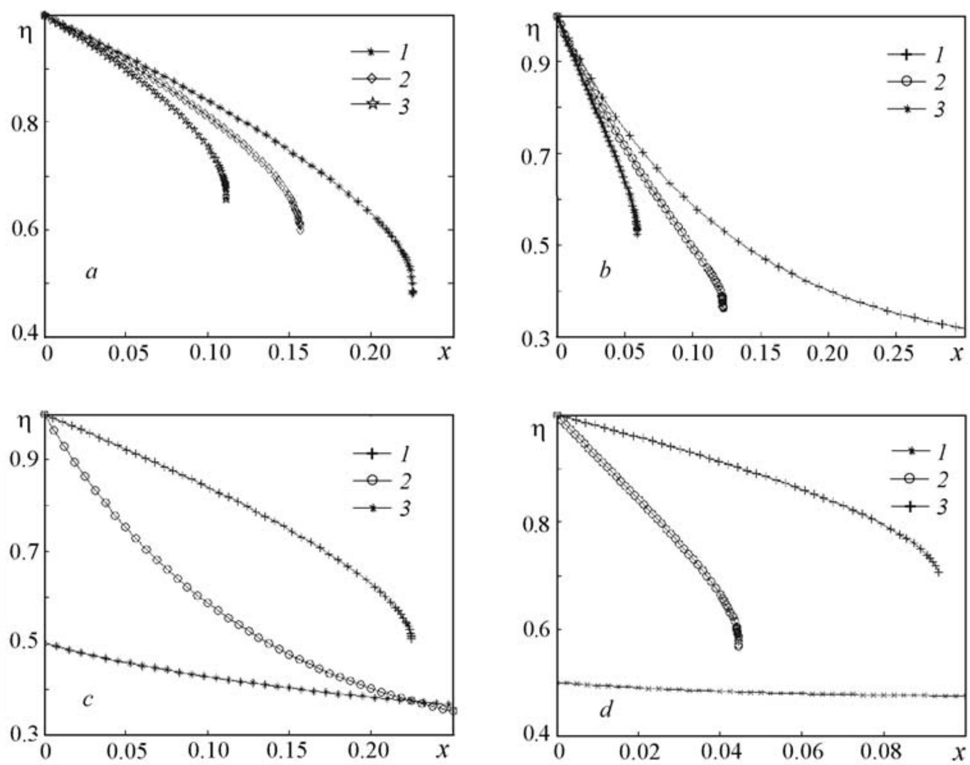

Fig. 2. Distribution of the function $\eta = \eta(x)$, $x \geq 0$: a) $\varphi_0 = 0$; $\beta_{H_1} = 0$ (1), 0.05 (2), 0.1 (3); b) $\varphi_0 = 0.5$; $\beta_{H_1} = 0$ (1), 0.05 (2), 0.1 (3); c) $\beta_{H_1} = 0$; $\varphi_0 = 0$ (1), 0.5 (2), 1 (3); d) $\beta_{H_1} = 0.125$; $\varphi_0 = 0$ (1), 0.5 (2), 1 (3). At $\beta_{H_1} = 0$, $\varphi_0 = 1$, and $\beta_{H_1} = 0.125$, $\varphi_0 = 1$ at the running wave front $x = 0$, the function $\eta = \eta(x)$ has a strong discontinuity. In the other cases, $\eta(0) = 1$.

As mentioned above, the bound of existence of a solution of the type of running waves in gas dynamics at $\varphi_0 = 0$ is characterized by the condition $\left.\frac{\partial \eta}{\partial x}\right|_{x=x_1} = \infty$. The values of the variables $x = x_1$, $\eta = \eta_1(x_1)$ are determined therewith by the equality of the wave front velocity and the velocity of sound [10–13]. In magnetic hydrodynamics, the existence of running waves on the finite interval of change in the independent variables is also associated with sound waves.

But if, as in the case under consideration, the magnetic field strength has only one component transverse to the direction of motion, then there exists a so-called fast magnetic wave. Its mass velocity can be determined by the formula [15]

$$C_+ = \sqrt{C^2 + C_H^2},$$

where $C = \rho\sqrt{RT}$; $C_H = \rho\frac{H}{\sqrt{4\pi\rho}}$. Assume that

$$D^2 = C_+^2 = \rho^2 RT + \rho\,\frac{H^2}{4\pi}. \tag{66}$$

Using the change of variables (29), we represent (66) in the form

$$1 = \eta^{-2} f + \eta^{-1} h^2. \tag{67}$$

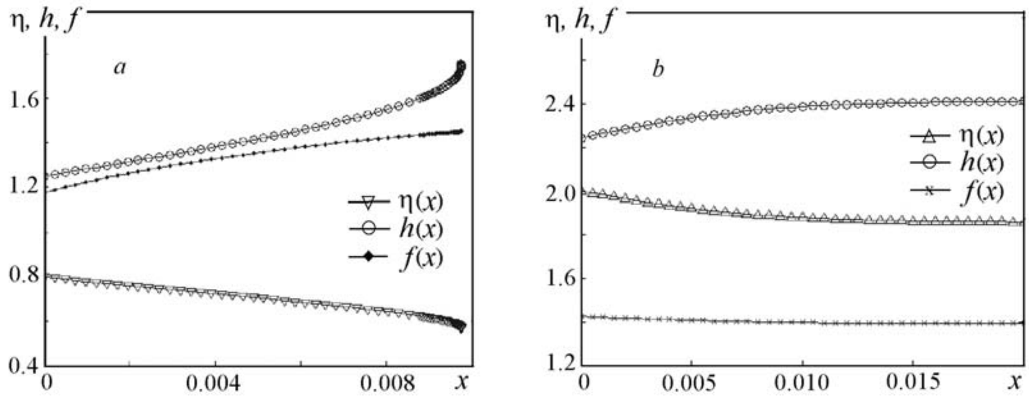

Fig. 3. Distribution of the functions $\eta = \eta(x)$, $h = h(x)$, and $f = f(x)$ at $\beta_{H_1} = 0.05$: a) $\varphi_0 = 0.168$; b) $\varphi_0 = 0.472$. At the running wave front $x = 0$ the sought quantities have a strong discontinuity at arbitrary φ_0 and β_{H_1}.

Let us further represent in (67) the expression of the functions $f = f(x)$ and $h = h(x)$ from (37) at $f_1 = 0$ taking into account that $h_1^2 = 8\pi\beta_{H_1}$. Manipulations yield

$$\eta^2 (1 - 2\eta) + \beta_{H_1} (1 + \eta^2) = 0 . \qquad (68)$$

From (63) it follows that in the case of $\varphi_0 = 0$ the derivative $\frac{d\eta}{dx}$ becomes infinite at condition (68), i.e., at equality of the velocity D and the velocity of the magnetic sound wave C_+.

Figure 3 illustrates the distribution of the dimensionless functions of the specific volume $\eta = \eta(x)$, the temperature $f = f(x)$, and the magnetic field strength $h = h(x)$ at $f_1 = 0.1$ and indices $a_0 - a_1$, $b_0 = 1$, and $b_1 = -1$.

Since $a_0 > a_1$, the sought quantities at the running wave front have a strong discontinuity at any values of φ_0 and β_{H_1}.

Conclusions. A system of magnetic hydrodynamic equations taking into account the infinite conduction and relaxation of the heat flow (1)–(5) has been considered. Four families of system characteristics have been determined. The laws at the front of a strong discontinuity of magnetohydrodynamic and thermal waves have been formulated. The magnetic hydrodynamic equations have been solved by the running-wave method. It has been shown that for the given problem the stability condition of the strong discontinuity of magnetohydrodynamic and thermal quantities in the formulation of the theory of generalized solutions of quasi-linear equations is fulfilled: at $n = 4$, $n + 1 = 5$ characteristics arrive at a point on the discontinuity line and $n - 1 = 3$ characteristics leave it.

NOTATION

A, matrix of quasi-linear equations; $a_{0,1}$, $b_{0,1}$, exponents of the dependence of k and τ on the temperature and density; **b**, vector of the right side of the system of quasi-linear equations; C, isothermic velocity of sound; C_h, velocity of sound associated with the magnetic field strength; C_T, propagation velocity of thermal perturbations at $\tau \neq 0$; C_γ, adiabatic velocity of sound; D, running wave front velocity; D_M, mass velocity of the discontinuity front; E, unit matrix; f, dimensionless temperature; $F = F(m, t)$, general notation of the function of independent variables m, t; H, magnetic field; h, dimensionless magnetic field; k, mass heat conductivity coefficient; l, discontinuity coordinate for the mass variable m; m, Lagrange mass variable; M_0, constant with the coordinate dimension of m; m_{dis}, trace of the surface or discontinuity trajectory; p, material pressure; p_H, magnetic pressure; R, universal gas constant; T, temperature; t, Lagrange time; v, velocity; V, auxiliary function; $\tilde{V}$, dimensionless value of the function V; W, heat flow; x, dimensionless independent variable; α, dimensionless velocity; β, dimensionless pressure; β_H, dimensionless magnetic pressure; γ, adiabatic constant; ε, specific internal energy; η, dimensionless specific volume; λ, eigenvalues of A; ρ, medium density; τ, relaxation time of the heat flow; φ_0, dimensionless constant; ω, dimensionless flow; k_0, τ_0, constant factors in exponential expressions of the function k and τ; $\tilde{\eta}$, z, small values of $\eta(x)$ in the vicinity of $\eta = 1$ and $\eta = \eta_{\gamma_0}$, respectively. Subscripts: 2, quantities behind the discontinuity front; 1, before the front; dis, discontinuity.

REFERENCES

1. G. A. Moses and J. J. Duderstadt, Improved treatment of electron thermal conduction in plasma hydrodynamics calculation, *Phys. Fluids*, **20**, No. 5, 762–770 (1977).
2. V. I. Kosarev, E. I. Levanov, and E. N. Sotskii, *On a Technique for Describing the Process of Electron Heat Conductivity in a High-Temperature Plasma*, Preprint No. 142 of the M. V. Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, Moscow (1981).
3. E. I. Levanov and E. N. Sotskii, *Heat Transfer with Allowance for the Heat-Flow Relaxation. Mathematical Simulation. Nonlinear Differential Equations* [in Russian], Nauka, Moscow (1987).
4. E. M. Burkhanov, E. N. Lyutikov, and S. A. Medin, *Hyperbolic Heat Conductivity and the Second Law of Thermodynamics*, Preprint No. 2-462 of the Joint Institute for High Temperatures, Russian Academy of Sciences, Moscow (2002).
5. O. N. Shablovskii, *Relaxation Process in Nonlinear Media* [in Russian], GGTU im. Sukhogo, Gomel' (2003).
6. P. P. Volosevich, E. I. Levanov, and E. V. Severina, Mathematical simulation of heat transfer in a moving medium with allowance for the heat-flow relaxation and volume energy sources, *Izv. Vyssh. Uchebn. Zaved., Matematika*, No. 1 (512), 31–39 (2005).
7. P. P. Volosevich, E. I. Levanov, and E. V. Severina, Temperature shock waves in a moving medium with allowance for the relaxation of the heat flow, *Inzh.-Fiz. Zh.*, **79**, No. 4, 57–68 (2006).
8. P. P. Volosevich, E. I. Levanov, and E. V. Severina, Traveling-wave-type solutions with account for the hyperbolic heat transfer, *Inzh.-Fiz. Zh.*, **81**, No. 2, 290–302 (2008).
9. P. P. Volosevich, N. V. Zmitrenko, E. I. Levanov, and E. V. Severina, Dynamics and heating of a plasma with allowance for the heat-flow relaxation, *Mat. Modelir.*, **20**, No. 4, 57–68 (2008).
10. A. A. Samarskii, S. P. Kurdyumov, and P. P. Volosevich, Traveling waves in a medium with nonlinear heat conductivity, *Zh. Vych. Mat. Mat. Fiz.*, **5**, No. 2, 199–217 (1965).
11. P. P. Volosevich and E. I. Levanov, *Self-Similar Solutions of Gas Dynamics and Heat-Transfer Problems* [in Russian], MFTI, Moscow (1997).
12. P. P. Volosevich, E. I. Levanov, and S. A. Fetisov, *Self-Similar Solutions of Heating and Dynamics of Plasma* [in Russian], MFTI, Moscow (2001).
13. P. P. Volosevich, E. V. Ermolin, and E. I. Levanov, *Mathematical Simulation of the Gas-Dynamic Processes with Sources* [in Russian], MZ-Press, Moscow (2006).
14. B. L. Rozhdestvenskii and N. N. Yanenko, *Systems of Quasi-Linear Equations* [in Russian], Nauka, Moscow (1978).
15. A. A. Samarskii and Yu. P. Popov, *Difference Methods for Solving Problems of Gas Dynamics* [in Russian], Nauka, Moscow (1980).
16. A. N. Tikhonov and A. A. Samarskii, *Equations of Mathematical Physics* [in Russian], Izd. MGU, Moscow (1999).
17. E. I. Levanov and E. N. Sotskii, *On Traveling Waves in a Medium with Hyperbolic Heat Conductivity*, Preprint No. 193 of the M. V. Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, Moscow (1982).
18. L. Spitzer, *Physics of Fully Ionized Gases* [Russian translation], Mir, Moscow (1965).